Looking Ahead to Chapter 10

**Focus**
In Chapter 10, you will learn how to find the measure of angles, arcs, and segments in circles.

**Chapter Warmup**

Answer these questions to help you review skills that you will need in Chapter 10.

Complete the table. If necessary, write your answers in terms of \( \pi \).

<table>
<thead>
<tr>
<th></th>
<th>Radius</th>
<th>Diameter</th>
<th>Circumference</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>2 inches</td>
<td></td>
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<tr>
<td>2.</td>
<td>10 cm</td>
<td></td>
<td></td>
<td>81( \pi ) square feet</td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
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</tbody>
</table>

Simplify each expression completely. Write your answer as a fraction in simplest form.

4. \( \frac{1}{2}(75 - 32) \)
5. \( \frac{1}{2}(16 + 37) \)
6. \( \frac{60}{360}(32\pi) \)

Read the problem scenario below.
You collect coins.

7. The diameter of a quarter is 2.426 centimeters. What is the circumference of a quarter? What is the area of a quarter? Use 3.14 for \( \pi \) and round your answers to three decimal places.

8. The diameter of a nickel is 2.121 centimeters. What is the circumference of a nickel? What is the area of a nickel? Use 3.14 for \( \pi \) and round your answers to three decimal places.

**Key Terms**
circle \( \text{p. 391} \)
center \( \text{p. 391} \)
radius \( \text{p. 391, 413} \)
chord \( \text{p. 392, 407} \)
diameter \( \text{p. 392, 408} \)
secant \( \text{p. 392, 403} \)
tangent \( \text{p. 393, 405} \)
point of tangency \( \text{p. 393, 414} \)
central angle \( \text{p. 394} \)
inscribed angle \( \text{p. 394} \)
arc \( \text{p. 395, 408} \)
semicircle \( \text{p. 395} \)
minor arc \( \text{p. 395} \)
major arc \( \text{p. 395} \)
measure of a minor arc \( \text{p. 398} \)
intercepted arc \( \text{p. 399} \)
perpendicular bisector \( \text{p. 407} \)
tangent line \( \text{p. 413} \)
tangent segment \( \text{p. 414} \)
arc length \( \text{p. 418} \)
concentric \( \text{p. 421} \)
sector of a circle \( \text{p. 422} \)
segment of a circle \( \text{p. 424} \)
The New York City sewer system contains 6600 miles of pipes. If these pipes were placed end to end, they would reach from New York to Alaska and back. In Lesson 10.3, you will find the measures of angles formed on manhole covers.

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   Arc Length  p. 417

10.7 Playing Darts
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Riding a Ferris Wheel
Introduction to Circles

Objectives
In this lesson, you will:
- Identify parts of a circle.
- Draw parts of a circle.

Key Terms
- circle
- center
- radius
- chord
- diameter
- secant
- tangent
- point of tangency
- central angle
- inscribed angle
- arc
- semicircle
- minor arc
- major arc

SCENARIO
The first Ferris wheel was built in 1893 for the Chicago World’s Fair to rival the Eiffel Tower, which was built for the Paris World’s Fair. Today, the Sky Dream Fukuoka Ferris wheel in Japan is the world’s largest Ferris wheel.

Problem 1  Going Around and Around

A Ferris wheel is in the shape of a circle.

A circle is the set of all points in a plane that are equidistant from a point called the center. The distance from a point on the circle to the center is the radius of the circle. A circle is named by its center. For instance, the circle above is circle P.

Use the circle to answer the questions below.

A. Name the circle. Use a complete sentence in your answer.

B. For each of the points above, tell whether the point is on the circle, at the center, or inside the circle. Use a complete sentence in your answer.
Problem 1

Investigate Problem 1

1. Can you draw a line through the circle below so that it intersects the circle at exactly two points? If so, draw the line and plot the points. This line is called a secant of the circle.

How is a chord different from a secant? Use complete sentences to explain your reasoning.

C. Use a straightedge to draw $\overline{OB}$. Where are the endpoints located with respect to the circle? Use a complete sentence in your answer.

This segment is a radius of the circle. How many radii does a circle have? Use a complete sentence to explain your reasoning.

D. Use a straightedge to draw $\overline{AC}$. Then use a straightedge to draw $\overline{BD}$. How are the line segments different? How are they the same? Use complete sentences in your answer.

Both line segments are chords of the circle. Segment $\overline{AC}$ is called a diameter of the circle. Why isn’t $\overline{BD}$ a diameter? Use a complete sentence in your answer.

E. How does the length of the diameter of a circle relate to the length of the radius? Use a complete sentence in your answer.
Investigate Problem 1

2. Can you draw a line through the circle below so that it intersects the circle at exactly one point? If so, draw the line and plot the point. The line is called a **tangent** of the circle and the point is called the **point of tangency**.

![Circle](image)

Choose another point on the circle. How many tangents can you draw through the point? Use a complete sentence in your answer.

3. Complete the definitions below. Then draw a picture that demonstrates the definition.

   A **_____________** of a circle is a line segment in which both endpoints are on the circle.

   A **_____________** of a circle is a line that intersects the circle at two points.

   A **_____________** of a circle is a chord that passes through the center of the circle. It is also the distance across the circle through its center.
Investigate Problem 1

A ______________ of a circle is a segment that is drawn from the center of the circle to a point on the circle. It is also the distance from the center to a point on the circle.

A ______________ of a circle is a line that intersects the circle at exactly one point.

Problem 2  Sitting on the Wheel

Four friends are riding a Ferris wheel in the positions shown.

A. Draw an angle with the center of the Ferris wheel as its vertex so that Dru and Marcus are located on the sides of the angle. This angle is a central angle.

B. Draw an angle with Kelli as its vertex so that Marcus and Dru are located on the sides of the angle. This angle is an inscribed angle.

C. Draw an angle with Wesley as its vertex so that Dru and Marcus are located on the sides of the angle. This angle is also an inscribed angle.

D. How are these angles the same? How are they different? Use complete sentences in your answer.
1. A copy of the Ferris wheel from Problem 2 is shown below. Label the location of each person with the first letter of his or her name.

With your pencil, trace on the part of the circle that is between points $D$ and $M$. This portion of the circle is called an arc. An arc is an unbroken portion of a circle that lies between two points on the circle. An arc is named by its two endpoints. So, the arc you traced above is called arc $DM$ or arc $MD$. In symbols, you write this as $DM$ or $MD$.

2. Draw a diameter on the circle above so that point $D$ is an endpoint. Label the unknown endpoint as point $Z$. Then trace the arc that starts at point $D$, passes through point $K$ and ends at point $Z$. This arc is a semicircle.

Given a diameter of a circle, how many semicircles can be identified? Use a complete sentence in your answer.

Because this arc passes through point $K$, we call the semicircle described above semicircle $DKZ$ to distinguish it from the other semicircle that has points $D$ and $Z$ as its endpoints. Name this other semicircle.

3. A minor arc is an arc that is less than a semicircle. A major arc is an arc that is greater than a semicircle. Name one minor arc and one major arc in the circle above.

4. Minor arcs are named using two points. Major arcs are named using three points. Why do you think this is?
Objectives
In this lesson, you will:
■ Determine the measures of arcs.
■ Determine the measures of inscribed angles and central angles.

Key Terms
■ measure of a minor arc
■ intercepted arc

SCENARIO
Before airbags were installed in the steering wheels of cars, the recommended position for holding the steering wheel was the 10-2 position. Now, one of the recommended positions is the 9-3 position to account for the airbags. The numbers 10, 2, 9, and 3 refer to the numbers on a clock. So the 10-2 position means that one hand is at 10 o’clock and the other hand is at 2 o’clock.

Problem 1
Keep Both Hands on the Wheel
The circles below represent steering wheels, and the points on the circles represent the positions of a person’s hands.

A. For each circle, use the given points to draw a central angle. The hand position on the left is 10-2 and the hand position on the right is 11-1. What are the names of your central angles?

Without using a protractor, determine the measures of your central angles. Use complete sentences to explain how you found your answer.

B. How do the measures of these angles compare? Use a complete sentence in your answer.
Problem 1  Keep Both Hands on the Wheel

C. Why do you think the hand position represented by the circle on the left was recommended and the hand position represented on the right is not recommended? Use a complete sentence in your answer.

D. For each circle, name the minor arcs given by the points on the circle.

What are the central angles that are associated with each of these minor arcs?

Every minor arc has a central angle that it is associated with. The **measure of a minor arc** is the measure of its central angle. You can write the measure of a minor arc in symbols in the same way that you write the measure of an angle in symbols. For instance, if the measure of a minor arc $\overline{PQ}$ is $30^\circ$, you can write $m\overline{PQ} = 30^\circ$. What are the measures of the minor arcs you named in part (D)? Write your answers by using symbols.

E. Plot and label point $Z$ on each circle so that it does not lie between the endpoints of the minor arcs you identified in part (D). Find the measures of the major arcs that have the same endpoints as the minor arcs in part (D). Explain how you found your answer. Use a complete sentence in your answer.

F. What is the measure of a semicircle? Use a complete sentence to explain your reasoning.
Investigate Problem 1

1. Draw inscribed angle $\angle PSR$ on circle $Q$.

![Diagram of circle with inscribed angle PSR]

What is the arc that is in the interior of $\angle PSR$?

This arc is called an **intercepted arc** because it is formed by the intersections of the sides of the angle with the circle.

2. Consider the central angle that is shown below. Use a straight-edge to draw an inscribed angle that contains points $A$ and $B$ on its sides. Name the vertex of your angle point $P$.

![Diagram of circle with central angle]

What do the angles have in common? Use a complete sentence in your answer.

Use a protractor to measure the central angle and the inscribed angle. How is the measure of the inscribed angle related to the measure of the central angle and the measure of $\overline{AB}$? Use complete sentences in your answer.

Use a straightedge to draw a different inscribed angle that contains points $A$ and $B$ on its sides. Name its vertex point $Q$. Measure the inscribed angle. How is the measure of the inscribed angle related to the measure of the central angle and the measure of $\overline{AB}$? Use complete sentences in your answer.
Investigate Problem 1

Use a straightedge to draw one more inscribed angle that contains points $A$ and $B$ on its sides. Name its vertex point $R$. Measure the inscribed angle. How is the measure of the inscribed angle related to the measure of the central angle and the measure of $AB$? Use complete sentences in your answer.

3. Complete the following statement:
The measure of an inscribed angle in a circle is __________ the measure of its intercepted arc.

4. What can you conclude about inscribed angles that have the same intercepted arc? Use a complete sentence in your answer.

5. Use the given information to complete each statement.

- $m \angle PTQ = \underline{\hspace{2cm}}$
- $m \overline{PQ} = \underline{\hspace{2cm}}$
- $m \angle PRQ = 32^\circ$

- $m \angle XZY = \underline{\hspace{2cm}}$
- $m \angle PTQ = \underline{\hspace{2cm}}$
- $m \angle XWY = \underline{\hspace{2cm}}$

- $m \overline{XY} = 80^\circ$
10.3 Manhole Covers
Measuring Angles Inside and Outside of Circles

Objectives
In this lesson, you will:
■ Determine measures of angles formed by two chords.
■ Determine measures of angles formed by two secants.
■ Determine measures of angles formed by a tangent and a secant.
■ Determine measures of angles formed by two tangents.

Key Terms
■ secant
■ tangent

SCENARIO  Manhole covers are heavy removable plates that are used to cover up maintenance holes in the ground. Most manhole covers are circular and can be found all over the world. The tops of these covers can be plain or have beautiful designs cast into their tops.

Problem 1  Inside the Circle
Circle O below shows a simple manhole cover design.

A. Consider $\angle BED$. How is this angle different from the angles that you have seen so far in this chapter? How is this angle the same? Use complete sentences in your answer.

B. Can you determine the measure of $\angle BED$ with the information you have so far? If so, how? Use complete sentences to explain your reasoning.

C. Draw chord $CD$. Use the information given in the figure above to name the measures of any angles that you do know. Use complete sentences to explain how you found your answers.
Problem 1

Inside the Circle

D. How does \( \angle BED \) relate to \( \triangle CED \)? Use a complete sentence in your answer.

E. Complete the statement below that shows the relationship between the measures of \( \angle BED \), \( \angle EDC \), and \( \angle ECD \).

\[ m\angle EDC + \underline{\quad} = \underline{\quad} \]

F. What is the measure of \( \angle BED \)? Use a complete sentence in your answer.

Investigate Problem 1

1. Consider circle \( P \) shown below. Draw chord \( XY \) on the figure below.

   \[ \begin{array}{c}
   V \\
   P \\
   W \\
   X \\
   Z \\
   Y
   \end{array} \]

   Write an expression for \( m\angle WXY \) in terms of \( m\overline{WY} \).

   Write an expression for \( m\angle VYX \) in terms of \( m\overline{VX} \).

   Write an expression for \( m\angle WZY \) in terms of \( m\overline{WY} \) and \( m\overline{VX} \).

   The following statement summarizes what you discovered.

   The measure of an angle formed by two intersecting chords is half of the sum of the measures of the arcs intercepted by the angle and its vertical angle.

   Consider \( \angle WZY \) again. What is the arc that is intercepted by \( \angle WZY \)?
Investigate Problem 1

Name the angle that is vertical to $\angle WZY$. Then name the arc that is intercepted by the angle vertical to $\angle WZY$.

2. Consider the figure below. By the statement on the previous page, we can write that $\angle CEB = \frac{1}{2}(m\overset{⏜}{CB} + m\overset{⏜}{AD})$. Write similar expressions for $m\angle AED$, $m\angle AEC$, and $\angle DEB$.

Problem 2

Outside the Circle

Circle $T$ shows another simple manhole cover design.

$mKM = 80^\circ$  
$mLN = 30^\circ$

A. Consider $KL$ and $MN$. Use a straightedge to draw secants that coincide with each segment. Where do the secants intersect? Use a complete sentence in your answer. Label this point as point $P$ on the figure above.

B. Draw chord $KN$. Can you determine the measure of $\angle KPM$ with the information you have so far? If so, how? Use complete sentences to explain your reasoning.
Problem 2  **Outside the Circle**

C. Use the information given in the figure on the previous page to name the measures of any angles that you do know. Use complete sentences to explain how you found your answers.

D. How does $\angle KPN$ relate to $\triangle KPN$? Use a complete sentence in your answer.

E. Complete the statement below that shows the relationship between the measures of $\angle KPN$, $\angle NKP$, and $\angle KNM$.

$$\text{________ } + m\angle NKP = \text{________}$$

F. What is the measure of $\angle KPN$? Use a complete sentence in your answer.

G. Describe the measure of $\angle KPM$ in terms of the measures of both arcs intercepted by $\angle KPM$. Use a complete sentence in your answer.

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**Investigate Problem 2**

1. The following statement summarizes what you discovered in Problem 2.

If an angle is formed by two intersecting secants so that the angle is outside the circle, then the measure of this angle is half of the difference of the measures of the arcs that are intercepted by the angle.

Consider the figure below. Use the statement above to write an expression for $m\angle CAD$ in terms of the measures of the arcs that are intercepted by $\angle CAD$. 

![Diagram of a circle with secants and angles labeled]
Investigate Problem 2

2. In circle $P$ below, the line through point $T$ is tangent to the circle and is perpendicular to $UT$.

What are the measures of $\angle UXT$ and $\angle UYT$? What are the measures of $\angle UTV$ and $\angle UTW$? Do you think that there is a relationship between $m\angle UXT$ and $m\angle UTV$? Do you think that there is a relationship between $m\angle UYT$ and $m\angle UTW$? If so, what is the relationship?

In circle $Q$ below, the line through points $T$ and $S$ is tangent to the circle.

Use a straightedge to draw the central angle that is associated with $RT$. Then use your protractor to measure $\angle RQT$ and $\angle RTS$. How do the measures of the angles compare? Use a complete sentence in your answer.

Complete the following statement:
The measure of an angle that is formed by a tangent and a chord is __________ the measure of the arc intercepted by the chord.
Investigate Problem 2

3. Suppose that a tangent and a secant to a circle intersect, as shown below. Draw a chord that connects point Q and point T on the circle. Then use an argument similar to the one in Problem 2 to show that \( m \angle QST = \frac{1}{2}(m\overarc{QT} - m\overarc{RT}) \).

![Diagram of a circle with a tangent and a secant intersecting, showing points Q, R, S, T, and O.]

4. Suppose that two tangents to a circle intersect, as shown below. Draw a chord that connects point B and point D on the circle. Then use an argument similar to the one in Problem 2 to show that \( m \angle BCD = \frac{1}{2}(m\overarc{BD} - m\overarc{BGD}) \).

![Diagram of a circle with two tangents intersecting, showing points A, B, C, D, G, E, and O.]
Objectives
In this lesson, you will:
- Determine the relationships between a chord and a diameter of a circle.
- Determine the relationships between congruent chords and their minor arcs.

Key Terms
- chord
- diameter
- perpendicular bisector
- arc

SCENARIO
Color theory is a set of rules that are used to create color combinations. A color wheel is a visual representation of color theory. There are many kinds of color wheels; we will consider the RYB (red-yellow-blue) color wheel.

The color wheel is made of three different kinds of colors: primary, secondary, and tertiary. Primary colors (red, blue, and yellow) are the colors you start with. Secondary colors (orange, green, and purple) are created by mixing two primary colors. Tertiary colors (red-orange, yellow-orange, yellow-green, blue-green, blue-purple, red-purple) are created by mixing a primary color with a secondary color.

Problem 1
Mixing Primary Colors

On the circle below, the locations of the primary colors on the color wheel are points Y (yellow), R (red), and B (blue).

A. Use a straightedge to draw a chord that has endpoints that are primary colors. What color is created if you mix these two colors?

B. Use your compass and straightedge to draw the perpendicular bisector of the chord.

C. What do you notice about your perpendicular bisector? Use a complete sentence in your answer.

Take Note
You drew the perpendicular bisector of a line segment with a compass and straightedge in Lesson 5.6.
**Problem 1**  Mixing Primary Colors

**D.** Complete the following statement:

The perpendicular bisector of a chord passes through the ____________ of the circle.

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**Investigate Problem 1**

1. On circle $T$ below, draw a chord $AB$ that does not pass through the center of the circle. Then use the following steps to draw a diameter that is perpendicular to your chord.

Place your compass point on the center of the circle. Draw an arc that intersects the chord at two points. Name these points $C$ and $D$. Now open your compass wider than half the distance between $CD$. Place the point of the compass on point $C$ and draw an arc towards the center of circle. Place the point of the compass on point $D$ and draw an arc towards the center of the circle. Use your straightedge to draw the diameter that passes through the intersection of the arcs.

Label the point where the diameter intersects the chord as point $P$. Label the point where the diameter intersects $AB$ as point $Q$.

How does the length of $AP$ compare to the length of $PB$? What does this tell you about the diameter? Use a complete sentence in your answer.

How does the measure of $AQ$ compare to the measure of $BQ$? Use complete sentences to explain your reasoning.

Complete the following statement:

If a diameter of a circle is perpendicular to the chord, then the diameter __________ the chord and its arc.
Investigate Problem 1

2. What does $TP$ represent in the relationship between point $T$ and chord $AB$? Use a complete sentence in your answer.

3. Use a straightedge to draw two congruent chords, chord $AB$ and chord $CD$ on the circle below so that they are not parallel. The chords should not be diameters.

For each chord, use your compass and straightedge to draw a line segment that represents the distance from the center of the circle to the chord. Then use your compass to compare the lengths of these segments. What do you notice? Use a complete sentence in your answer.

Complete the following statement:
Congruent chords are the ________________ from the center of the circle.

4. Draw two chords on the circle below so that they are not parallel. Then use the chords to locate the center of the circle. Use complete sentences to explain how you found the center of the circle.
Problem 2  Mixing Primary and Secondary Colors

On the circle below, the locations of the primary colors on the color wheel are points $R$ (red), $Y$ (yellow), and $B$ (blue), and the locations of the secondary colors are points $O$ (orange), $G$ (green), and $P$ (purple).

A. Use a straightedge to draw two congruent chords so that the chords are not diameters and so that one endpoint is a primary color and one endpoint is a secondary color. You can use a compass to verify that the chords are the same length. Write the names of your chords below. Identify the colors that would be created if you mixed the colors of the endpoints of each chord.

B. From each endpoint of each chord, use your straightedge to draw a radius. Name the central angle formed by each pair of radii. Use a protractor to find the measures of these central angles. What do you notice? Use a complete sentence in your answer.

C. What does part (B) tell you about the minor arcs formed by the chords? Use a complete sentence to explain your reasoning.

D. Complete the following statement:
   If two chords in a circle are congruent, then their __________ are congruent.
Investigate Problem 2

1. The statement in part (D) is a conditional statement. Write the converse of the statement in part (D). Do you think that this statement is true?

2. Consider the circle below. Suppose that $AB \equiv CD$.

   \[
   \begin{array}{c}
   A \\
   D \\
   B \\
   C \\
   P
   \end{array}
   \]

   Use a straightedge to draw $AP, BP, CP,$ and $DP$. How do the lengths of the segments compare? Use a complete sentence to explain your reasoning. Mark this information on the figure above.

   How does $m\angle APB$ compare to $m\angle CPD$? Use a complete sentence to explain your reasoning. Mark this information on the figure above.

   What can you conclude about $\triangle APB$ and $\triangle CPD$? Explain. Use a complete sentence in your answer.

   What does this tell you about chord $AB$ and chord $CD$? Use a complete sentence in your answer.

3. Complete the following statement:

   If two minor arcs in a circle are congruent, then their __________ are congruent.
**Objectives**

In this lesson, you will:
- Determine the relationship between a tangent line and a radius.
- Determine the relationship between congruent tangent segments.

**Key Terms**
- tangent line
- radius
- point of tangency
- tangent segment

**SCENARIO**

Total (solar) eclipses occur when the moon passes between Earth and the sun. The position of the moon creates a shadow on the surface of Earth.

A pair of tangent lines forms the boundaries of the *umbra*, the darker part of the shadow. Another pair of tangent lines forms the boundaries of the *penumbra*, the lighter part of the shadow.

**Problem 1** Blocking Out the Sun

Consider the tangents shown below.

A. For the sun and moon, use a straightedge to draw a radius from the center of the body to one of the tangents.

B. Use your protractor to measure the angle between each radius and tangent. What do you notice? Use a complete sentence in your answer.

C. Complete the following statement:

A tangent to a circle is ________________ to the radius that is drawn to the point of tangency.

**Investigate Problem 1**

1. What do you notice about the tangents that form the umbra in the first figure? Use a complete sentence in your answer.
Investigate Problem 1

2. Choose a point outside the circle below. Label this point $P$.

Use your straightedge to draw a line that passes through point $P$ so that it is tangent to circle $O$. Now use your straightedge to draw a different line that passes through point $P$ so that it is tangent to circle $O$. Label the points of tangency as points $Q$ and $R$.

Draw radius $OQ$ and draw radius $OR$. What are the measures of $\angle OQP$ and $\angle ORP$? How do you know? Use a complete sentence in your answer. Then mark this information on the figure above.

How does $OQ$ compare to $OR$? How do you know? Use a complete sentence in your answer. Then mark this information on the figure above.

Draw $\overline{OP}$. What is the relationship between $\triangle POQ$ and $\triangle POR$? Explain your reasoning. Use complete sentences in your answer.

What can you conclude about $\overline{PQ}$ and $\overline{PR}$? The segments are called tangent segments. Use a complete sentence to explain your answer.

3. Complete the following statement:

If two tangent segments to a circle are drawn from the same point outside the circle, then the tangent segments are ________________.
Investigate Problem 1

4. In the figure, $\overrightarrow{KP}$ and $\overrightarrow{KS}$ are tangent to circle $W$ and $m\angle PKS = 46^\circ$. Find $m\angle KPS$. Show all your work and use complete sentences to explain how you found your answer.

5. In the figure, $\overrightarrow{PS}$ is tangent to circle $M$ and $m\angle SMO = 119^\circ$. Find $m\angle MPS$. Show all your work and use complete sentences to explain how you found your answer.
Objective
In this lesson, you will:
■ Find the length of an arc of a circle.

Key Term
■ arc length

SCENARIO  Gears are used in many mechanical devices to provide torque, or the force that causes rotation. For instance, an electric screwdriver contains gears. The motor of an electric screwdriver can make the spinning components spin very fast, but the gears are needed to provide the force to push a screw into place. Gears can be very large or very small, depending on their application. Often gears work together, such as the gears below.

Problem 1  Large and Small Gears
Consider the circles below that model two gears that work together.

A. Use your protractor to draw a central angle on each circle that has a measure of 60°.

B. What is the measure of each of the minor arcs associated with these central angles? Use a complete sentence in your answer.

C. For each circle, the minor arc is what fraction of the circle? Use complete sentences to explain how you found your answer.
D. When gears are used together, the **circumferences** of the gears are important because the gears move together. Use a complete sentence to describe the circumference of a circle.

Which gear in Problem 1 has a greater circumference? Use a complete sentence to explain your reasoning.

E. What fraction of the circumference do you think is taken up by the minor arcs described on the previous page? Explain your reasoning. Use a complete sentence in your answer.

A portion of the circumference of a circle is called an **arc length**.

F. Suppose that the circumference of circle A is $48\pi$ inches and the circumference of circle B is $36\pi$ inches. What is the length of the minor arcs? Show all your work and use a complete sentence in your answer.

G. Consider the minor arcs of the central angles you drew. How do the **measures** of the arcs compare? How do the **lengths** of the arcs compare? Use complete sentences in your answer.

H. How is the **measure** of an arc different from the **length** of an arc? Use a complete sentence in your answer.
Lesson 10.6
Arc Length

Investigate Problem 1

1. Use complete sentences to explain how you can find the length of an arc when you know its measure.

2. Consider the circle below. What is the circumference of the circle? Leave your answer in terms of \( \pi \) and use a complete sentence in your answer.

Choose two points on the circle and label the points as point \( A \) and point \( B \). Then use a protractor to find the measure of \( \overarc{AB} \). \( \overarc{AB} \) is what fraction of the circle? Use a complete sentence in your answer.

Use your answers above to write an expression for the arc length of \( \overarc{AB} \). Leave your answers in terms of \( \pi \). Use a complete sentence in your answer.

3. Use the figure to complete the following statement:

The arc length of \( \overarc{AB} \) is \( \frac{m\overarc{AB}}{360^\circ} \cdot \)
Investigate Problem 1

4. Find the arc length of each circle below. Show all your work and leave your answer in terms of $\pi$.

5. Use complete sentences to describe the relationship between the measure of an arc and its arc length.
**Objective**

In this lesson, you will:
- Determine areas of sectors and segments of circles.

**Key Terms**
- concentric
- sector of a circle
- segment of a circle

**Scenario**

A standard dartboard is shown at the right. Each different section of the board is surrounded by wire and the numbers indicate scoring for a game. There are different games with different scoring that can be played on a dartboard, but the highest score from a single throw occurs when a dart lands at the very center, or bullseye, of the dartboard.

**Problem 1** Hitting the Bullseye

The dartboard is made of concentric circles that are divided into sections. Two circles are concentric if they have the same center and different radii.

A. The first circle inside the outermost circle of the dartboard has a diameter of 170 millimeters. Find the area of this circle. Show all your work and leave your answer in terms of $\pi$.

B. Imagine that the pie-shaped sections extend to the center of the circle. How many pie-shaped sections is this circle divided into? Use a complete sentence in your answer.

C. What is the measure of the central angle formed by one of these pie-shaped sections if all of the sections are congruent? Use a complete sentence to explain how you found your answer.

D. What is the measure of the minor arc associated with this central angle? Use a complete sentence in your answer.
Problem 1  Hitting the Bullseye

E. What fraction of the circle is the minor arc? Use a complete sentence in your answer.

F. What fraction of the circle’s area is covered by one of the pie-shaped sections? Use a complete sentence to explain how you found your answer.

How does this fraction compare to the fraction in part (E)?

G. What is the area of one pie-shaped section? Show all your work and use a complete sentence in your answer. Leave your answer in terms of $\pi$.

Investigate Problem 1

1. In Problem 1, how can the sides of the pie-shaped section be described with respect to the circle? Use a complete sentence in your answer.

Just the Math: Sector of a Circle  This pie-shaped section is called a sector of a circle. A sector of a circle is a portion of a circle that is bounded by two radii and the arc that is intercepted by the radii. Draw and shade in a sector on the circle below. Identify the points that form the sector on the circle below. Then list the names of the radii and the arc that is intercepted by the radii that form the sector.
2. Use complete sentences to explain how you can find the area of a sector when you know the measure of the arc that is intercepted by two radii and the radius of the circle.

3. Use the figure to complete the following statement:

The area of sector $AOB$ is $\frac{\text{mAB}}{360^\circ} \cdot \text{_____.}$

4. Consider the dartboard again. The innermost circle that is divided into 20 sectors has a diameter of 108 millimeters. Half of the sectors are one color and the other half are a different color. Use complete sentences to describe two different ways in which you could find the total area of half of the sectors. Then find this area and the area of one sector.
5. A segment of a circle is the portion of the circle that is bounded by a chord and an arc formed by the chord. In circle $C$, $\overline{AB}$ and $\overline{AB}$ form a segment of circle $C$. The radius of circle $C$ is 8 centimeters and $m\angle ACB = 90^\circ$.

How do you think that you can find the area of this segment? Use complete sentences in your answer.

Find the area of sector $ACB$. Show all your work and use a complete sentence in your answer. Leave your answer in terms of $\pi$.

Then find the area of $\triangle ACB$. Show all your work and use a complete sentence in your answer.

Find the area of the segment of the circle. Show all your work and use a complete sentence in your answer. Use 3.14 for $\pi$. 